Abduction Explained

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Claim: abduction is dual defeasible entailment.

Questions:

- What is defeasible entailment?
- What is dual defeasible entailment?
- Why do we think this formalises abduction?

Defeasible entailment $\mid\!\sim$

Light-Fan System:

- atomic facts: p = the light is on, q = the fan is on
- states = truth assignments: $S = \{11, 10, 01, 00\}$

Example of preference relation:

11	
10	01
00	

Default rule: components are normally on.

Now $p \mid\!\sim q$ since $PMod(p) \subseteq Mod(q)$

where $PMod(p) = \{11\}$ and $Mod(q) = \{11, 01\}$.

Dual defeasible entailment $\mid\!\sim^*$

$\models$ is given by $Mod(\alpha) \subseteq Mod(\beta)$.

$\mid\!\sim$ shrinks the lefthand side: $PMod(\alpha) \subseteq Mod(\beta)$

$\mid\!\sim^*$ inflates the right side: $Mod(\alpha) \subseteq QMod(\beta)$.

Def: $\alpha \mid\!\sim^* \beta$ iff $Mod(\alpha) \subseteq S - PMod(\neg\beta)$.

Inflate $Mod(\beta)$ by adding extraordinary models of $\neg\beta$.

	11	
10		01
	00	

$\neg(p \leftrightarrow q) \mid\!\sim^* \neg q$ since $\{10, 01\} \subseteq \{00, 10\} \cup \{01\}$

but it is here not the case that $\neg(p \leftrightarrow q) \mid\!\sim \neg q$

since $PMod(\neg(p \leftrightarrow q)) = \{10, 01\} \not\subseteq Mod(\neg q)$.

Duality, $\mid\!\sim$ and $\mid\!\sim^*$

1. $\mid\!\sim^*$ is not the converse of $\mid\!\sim$

   since $\neg p \mid\!\sim q$ but not $q \mid\!\sim^* \neg p$.

2. $\mid\!\sim^*$ is the dual of $\mid\!\sim$

   where the operation $()^*$ is defined by

   $(\neg\beta, \neg\alpha) \in R^*$ iff $(\alpha, \beta) \in R$.

   To see this, observe that

   - $\neg\beta \mid\!\sim^* \neg\alpha$ iff $\alpha \mid\!\sim \beta$

     since $PMod(\alpha) \subseteq S - Mod(\neg\beta)$

     iff $Mod(\neg\beta) \subseteq S - PMod(\neg\neg\alpha)$

   - $\neg\neg\alpha \mid\!\sim^{**} \neg\neg\beta$ iff $\alpha \mid\!\sim \beta$

Unlike defeasible entailment $\mid\!\sim$, classical entailment $\models$ is self-dual by contraposition: $\alpha \models \beta$ iff $\neg\beta \models \neg\alpha$.

Properties of $\mid\!\sim$ and $\mid\!\sim^*$

Kraus, Lehmann and Magidor list properties for $\mid\!\sim$, e.g.

Right Weakening (RW) $$\frac{\alpha \mid\!\sim \beta \quad \models \beta \rightarrow \gamma}{\alpha \mid\!\sim \gamma}$$

Cautious Monotonicity $$\frac{\alpha \mid\!\sim \beta \quad \alpha \mid\!\sim \gamma}{\alpha \wedge \beta \mid\!\sim \gamma}$$

Cut (Cautious LW) $$\frac{\alpha \wedge \beta \mid\!\sim \gamma \quad \alpha \mid\!\sim \beta}{\alpha \mid\!\sim \gamma}$$

which all have dual versions that hold for $\mid\!\sim^*$:

Monotonicity (LS) $$\frac{\beta \mid\!\sim^* \gamma \quad \models \alpha \rightarrow \beta}{\alpha \mid\!\sim^* \gamma}$$

Cautious RW $$\frac{\alpha \mid\!\sim^* \gamma \quad \beta \mid\!\sim^* \gamma}{\alpha \mid\!\sim^* \beta \vee \gamma}$$

Cautious RS $$\frac{\alpha \mid\!\sim^* \beta \vee \gamma \quad \gamma \mid\!\sim^* \beta}{\alpha \mid\!\sim^* \beta}$$

Both $\mid\!\sim$ and $\mid\!\sim^*$ satisfy Reflexivity, And, Or, Left and Right Logical Equivalence.

Algebraic roles of $\mid\!\sim$ and $\mid\!\sim^*$

Consider the Lindenbaum-Tarski algebra of propositions, with order relation $\models$, $\bot$ given by the equivalence class of contradictions, and $\top$ the class of tautologies.

For a fixed premiss α, the set $\{\beta : \alpha \mid\!\sim \beta\}$ is a *filter*, i.e. closed under $\wedge$ and $\models$.

But for a fixed consequence β, the set $\{\alpha : \alpha \mid\!\sim \beta\}$ of premisses merits no acclamation: it is not an *ideal* because $\mid\!\sim$ is nonmonotonic, so that $\alpha \mid\!\sim \beta$ does not always ensure that $\alpha \wedge \gamma \mid\!\sim \beta$, hence downward closure fails.

However, if we use the dual relation $\mid\!\sim^*$, then for a fixed consequence β the set $\{\alpha : \alpha \mid\!\sim^* \beta\}$ of premisses does form an ideal (although the set of consequences for a fixed α does not form a filter).

Abduction

CS Peirce proposed that abduction had the following 'perfectly definite logical form':

Premiss 1: The (possibly surprising) fact β is observed.

Premiss 2: If α were the case, β would follow as a matter of course.

Conclusion: Hence there is reason to suspect that α may be true.

Traditionally, premiss 2 was taken to mean $\alpha \models \beta$.

Some have loosened this to $\alpha \mid\sim \beta$.

We wish to replace premiss 2 by $\alpha \mid\sim^* \beta$.

Why? Because α is supposed to *explain* β.

Explanation

Criterion for "α is a potential partial explanation for β"?

$\alpha \models \beta$? No: take $q =$ that thing flies, $p =$ that thing is a bird. Then $p \not\models q$. (But $p \mid\!\sim q$. Hmmm.)

$\alpha \mid\!\sim \beta$? No: it is possible to have $\alpha \mid\!\sim \beta$ while all but the most preferred models of α are in fact typical (= maximally preferred) models of $\neg\beta$.

Hence let us require that $Mod(\alpha) \cap PMod(\neg\beta) = \emptyset$. Thus $\alpha \mid\!\sim^* \beta$ is precisely the criterion we seek.

Example: Think of the Light-Fan System as a nuclear powerplant. And suppose that q is observed.

11	$p \mid\!\sim q$?	Yes, $PMod(p) \subseteq Mod(q)$
01	$\neg p \mid\!\sim q$?	Yes, $PMod(\neg p) \subseteq Mod(q)$
00	$p \mid\!\sim^* q$?	Yes, $Mod(p) \subseteq S - PMod(\neg q)$
10	$\neg p \mid\!\sim^* q$?	No, $Mod(\neg p) \not\subseteq S - PMod(\neg q)$

Induction

CS Peirce suggested that there are 3 'elementary kinds of reasoning': deduction, abduction, and induction.

Traditionally, induction is for (universal) *rule-formation*:

Premiss: Robins use serotonin as a neurotransmitter.
Conclusion: All birds use serotonin as a neurotransmitter.

Or for *prediction*:

Premiss: Robins use serotonin as a neurotransmitter.
Conclusion: Doves use serotonin as a neurotransmitter.

Universal sentences represent universal rules. But outside mathematics, default rules are more important, represented by preference relations, not object-language sentences. So the only kind of induction to constrain via a semantic relation on sentences is prediction. Clearly $\mid\sim$ is the right sort of relation.

Category abduction and induction

Categorisation is an important part of thought, especially for coping with novelty and making predictions.

Two psychologically important measures:

- cue validity — probability that an object x is in a category C, given that x has features F
- category validity — probability that item x has features F, given that x is in category C.

Cue validity is analogous to abduction as constrained by $\mid\!\sim^*$, for high cue validity explains features F by category membership, and if x is in category C then x will not be a typical member of a contrast (non-C) category.

Category validity is analogous to induction as constrained by $\mid\!\sim$, for if x is in category C, then x will typically have features F.

Other approaches to abduction

Aliseda, Gabbay: abstract schema involving a relation R on sentences that may be interpreted variously; schema involves an explanandum E, background knowledge K, explanatory hypothesis H, and conditions such as $(K, E) \notin R$ and $(K * H, E) \in R$.

Compatible. Can accommodate $R = \mid\!\sim^*$.

Flach: Rationality principles. View of induction close to our $\mid\!\sim$. View of abduction essentially takes the 'explanatory consequence relation' to be the converse (!) of 'some consequence relation $\mid\!\sim$' although the nature of $\mid\!\sim$ is here left open (could be $\models$). In other words, α is supposed to explain β if $\alpha \mid\!\sim \beta$ or possibly if $\alpha \models \beta$.

Incompatible. We have already shown the flaws in this.